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# Boundary-Driven Cosmic Expansion: A Purely Geometric Theory of Inflation

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**Abstract**

Cosmic inflation remains the leading paradigm for explaining the earliest stage of cosmological evolution, yet its underlying physical mechanism continues to motivate alternative theoretical descriptions. In this work, we introduce a purely geometric framework in which cosmic expansion emerges from the collective evolution of an infinite ensemble of independently expanding universes represented as three-dimensional spheres embedded in a common Euclidean space. Unlike conventional inflationary models, the proposed formulation introduces neither scalar fields, spacetime curvature, vacuum energy, nor modified gravitational dynamics. Instead, expansion is governed exclusively by the availability of unconstrained boundary. The central concept of the theory is the *free boundary fraction*, a dimensionless geometric functional that measures the proportion of each universe's boundary remaining free from overlap with neighbouring universes. Beginning from a small set of geometric axioms, we derive a closed boundary-driven dynamical system, establish the existence, boundedness, monotonicity, and limiting properties of the free boundary functional, and show that the resulting constitutive law naturally yields an initial phase of maximum boundary-driven expansion followed by a progressive, self-regulated reduction as collective boundary interactions accumulate. The resulting dynamics provide a mathematically self-contained geometric mechanism that qualitatively reproduces several characteristic features commonly associated with the inflationary epoch while remaining fundamentally distinct from conventional exponential inflation. Rather than attributing primordial expansion to additional physical fields or phenomenological potentials, the proposed framework demonstrates that collective boundary geometry alone is sufficient to generate heterogeneous expansion histories and intrinsic self-regulation. This work establishes a new geometric perspective on early-universe evolution and provides a mathematical foundation for future theoretical development, numerical simulation, and observational investigation.

**Keywords:** Geometric cosmology; Boundary-driven expansion; Free boundary fraction; Geometric dynamical systems; Boundary interactions; Expanding universe ensembles

## 1 Introduction

One of the central objectives of modern cosmology is to explain the origin and early evolution of the observable universe. Within the standard cosmological framework, the hot Big Bang model successfully accounts for many large-scale observational phenomena, including the cosmic microwave background, the primordial abundance of light elements, and the large-scale distribution of matter Weinberg (2008); Kolb and Turner (1990); Dodelson and Schmidt (2020). Nevertheless, the classical Big Bang model alone does not naturally resolve several fundamental problems, including the horizon problem, the flatness problem, and the predicted abundance of topological relics such as magnetic monopoles Guth (1981).

Cosmic inflation was introduced as an elegant solution to these difficulties by postulating a brief epoch of extremely rapid primordial expansion prior to the conventional radiation-dominated era Guth (1981). Subsequent developments, including new inflation, chaotic inflation, and $R^2$ inflation, established inflation as the leading paradigm for describing the earliest stages of cosmological evolution Linde (1982); Albrecht and Steinhardt (1982); Linde (1983); Starobinsky (1980). Comprehensive treatments of the inflationary paradigm and its observational implications are provided by Mukhanov (2005); Liddle and Lyth (2000)In addition to resolving several conceptual problems of the classical Big Bang model, inflation provides a natural mechanism for generating the primordial density fluctuations that subsequently evolved into the observed large-scale structure of the universe Guth and Pi (1982). Modern observational studies continue to place increasingly stringent constraints on inflationary scenarios while leaving open the question of the precise physical mechanism responsible for the inflationary epoch Finelli et al. (2018); Baumann and Green (2023); Ellis et al. (2018).

Despite its remarkable phenomenological success, the microscopic origin of inflation remains an active subject of theoretical investigation. Most existing models introduce additional dynamical degrees of freedom, typically in the form of scalar fields and their associated potentials, to generate and terminate the inflationary phase Tsujikawa (2003). Alternative proposals have also explored different physical mechanisms capable of producing an early period of rapid cosmic expansion without relying exclusively on conventional inflaton dynamics Barua (2023). The continuing diversity of such approaches suggests that the mathematical principles underlying primordial expansion have not yet been uniquely established. Numerous reviews have emphasized that a large landscape of inflationary realizations remains compatible with current observations despite increasingly stringent experimental constraints Lyth and Riotto (1999); Bassett et al. (2006); Martin et al. (2014).

Motivated by this observation, the present work investigates whether certain qualitative features commonly associated with the early expansion history of the universe can arise from geometry alone. Rather than introducing additional physical fields, gravitational dynamics, or phenomenological potentials, we construct a purely geometric framework based on an infinite ensemble of expanding three-dimensional spheres embedded in Euclidean space. Within this framework, each universe evolves exclusively through the geometry of its boundary, while interactions arise solely through local boundary intersections. The resulting dynamics are governed by a single dimensionless quantity—the free boundary fraction—which measures the proportion of boundary remaining geometrically unconstrained.

The proposed model is intentionally formulated independently of General Relativity and conventional inflationary dynamics. Accordingly, the mathematical development presented in this paper should not be interpreted as a replacement for the standard cosmological paradigm. Instead, it establishes a self-contained geometric dynamical system whose qualitative behaviour exhibits an initial phase of maximum boundary-driven expansion followed by a self-regulated reduction in expansion arising from collective geometric constraints. This transition is subsequently interpreted as a possible geometric analogue of the inflationary transition, while remaining mathematically distinct from conventional exponential inflation.

The principal contribution of this paper is therefore the introduction of a boundary-driven geometric theory of cosmic expansion in which the evolution of each universe is determined entirely by the availability of unconstrained boundary. Beginning from a small set of geometric axioms, we derive a closed dynamical system, establish its principal mathematical consequences, and discuss its possible cosmological interpretation. Whether this boundary-driven perspective represents a complementary description of early-universe dynamics or the basis of a broader cosmological framework remains an open question for future theoretical and observational investigation.

## 2 Literature Review

The inflationary paradigm occupies a central position in modern cosmology as the prevailing explanation for the earliest stages of cosmic evolution. Since its original formulation by Guth as a mechanism for resolving the horizon, flatness, and monopole problems of the classical Big Bang model Guth (1981), inflation has developed into a broad theoretical framework encompassing numerous realizations based upon distinct physical assumptions. Subsequent developments, including new inflation Linde (1982); Albrecht and Steinhardt (1982), chaotic inflation Linde (1983), and the geometric $R^2$ inflationary model of Starobinsky Starobinsky (1980), demonstrated that a sufficiently rapid early expansion naturally explains several otherwise puzzling features of the observable universe while simultaneously generating the primordial fluctuations responsible for the formation of large-scale structure Guth and Pi (1982).

These developments collectively established inflation as the standard paradigm for early-universe cosmology Mukhanov (2005); Liddle and Lyth (2000).

Over the past four decades, increasingly precise cosmological observations have substantially constrained inflationary models without uniquely determining the physical mechanism responsible for the inflationary epoch. Contemporary reviews emphasize that although observational evidence strongly supports an early period of accelerated expansion, numerous competing theoretical realizations remain compatible with existing measurements Baumann and Green (2023); Ellis et al. (2018); Finelli et al. (2018). Consequently, inflation is more appropriately viewed as a successful phenomenological paradigm than as a uniquely established microscopic theory. For comprehensive surveys of modern inflationary model building and observational constraints, see Lyth and Riotto (1999); Bassett et al. (2006); Martin et al. (2014). Most existing models introduce one or more scalar fields whose dynamics, together with suitably chosen potentials, govern both the initiation and termination of the inflationary phase Tsujikawa (2003). Alternative approaches continue to investigate different physical mechanisms capable of producing similar cosmological behaviour while avoiding some of the assumptions inherent in conventional inflaton models Barua (2023).

Despite their mathematical diversity, nearly all existing inflationary theories share a common conceptual structure. The universe is typically modelled as a single connected spacetime whose global expansion is determined by field equations, gravitational dynamics, or additional matter components. The resulting expansion history is therefore controlled by local dynamical variables such as scalar fields, curvature corrections, vacuum energy, or modified gravitational interactions. Within this broad class of theories, geometry generally responds to the underlying dynamics rather than acting as the primary dynamical agent.

This viewpoint is deeply rooted in General Relativity, where the spacetime geometry is determined by the Einstein field equations and the matter-energy content of the universe Hawking and Ellis (1973); Wald (1984); Misner et al. (1973).

The present work adopts a fundamentally different mathematical perspective. Rather than introducing additional physical fields or modifying gravitational dynamics, we investigate whether collective geometric interactions alone can generate a self-regulating expansion process. The proposed framework considers an infinite ensemble of simultaneously evolving universes, each represented as an expanding three-dimensional sphere embedded within a common Euclidean space. Every universe possesses a fixed geometric centre while its boundary expands according to a boundary-driven constitutive law. Interactions arise exclusively through local boundary intersections, without the introduction of long-range forces, spacetime curvature, matter fields, or inflationary potentials.

Although conceptually distinct from existing inflationary and multiverse scenarios, this perspective is motivated by continuing theoretical investigations into eternal inflation, universe creation, and cosmological multiplicity Garriga et al. (2007); Aguirre and Johnson (2007).

Within this geometric picture, simultaneous primordial expansion is not restricted to a single universe. Instead, the proposed framework considers an ensemble of independently evolving universes embedded within a common Euclidean space. Although conceptually distinct from existing inflationary and multiverse scenarios, this perspective is motivated by continuing theoretical investigations into eternal inflation, universe creation, and cosmological multiplicity Linde (1986); Borde and Vilenkin (1994); Borde et al. (2003); Vilenkin (1983). Instead, all universes are assumed to originate from independent initial configurations and undergo concurrent boundary evolution. The mathematical construction adopts a purely Euclidean embedding space and therefore deliberately avoids introducing spacetime curvature or Lorentzian geometry during the development of the model. Similar geometric viewpoints are standard in differential geometry and manifold theory Nakahara (2003); O'Neill (1983). Because the centres remain fixed while the radii evolve, neighbouring universes inevitably begin to intersect once their expanding boundaries satisfy the appropriate geometric contact conditions. These intersections partition each boundary into unconstrained and constrained regions, thereby reducing the fraction of boundary available for continued unconstrained expansion. Consequently, the instantaneous expansion history of each universe becomes an emergent collective property determined by the evolving geometry of the entire ensemble rather than by isolated pairwise interactions or externally prescribed physical fields.

An immediate consequence of this formulation is that different universes generally experience different expansion histories. In contrast to conventional inflationary cosmology, where expansion is typically governed by common field dynamics, the present framework attributes heterogeneous evolution entirely to local geometric interactions Mukhanov (2005); Lyth and Riotto (1999); Bassett et al. (2006). Since the extent of boundary overlap depends upon the local geometric environment surrounding each universe, the free boundary available for expansion need not evolve identically throughout the ensemble. Universes embedded within densely populated neighbourhoods accumulate geometric constraints more rapidly than those located in relatively sparse regions, naturally producing heterogeneous expansion rates without introducing additional dynamical variables. The proposed framework therefore predicts that differences in expansion histories arise from collective boundary geometry itself.

The principal objective of the present work is not to replace the standard inflationary paradigm nor to derive the Einstein field equations from first principles. Accordingly, the present work should be interpreted as a geometric alternative to existing dynamical descriptions rather than a modification of General Relativity itself Carroll (2004); Wald (1984); Hawking and Ellis (1973). Instead, the goal is to establish a mathematically self-contained geometric theory in which boundary availability constitutes the sole state variable governing expansion. Beginning from a small collection of geometric axioms, we derive a closed boundary-driven dynamical system and demonstrate that it naturally admits an initial phase of maximum unconstrained expansion followed by a progressive reduction in expansion as collective boundary constraints emerge. These features are subsequently interpreted as a qualitative geometric analogue of the transition commonly associated with the inflationary epoch while remaining mathematically distinct from conventional exponential inflation.

To the best of our knowledge, existing cosmological models predominantly formulate primordial expansion through gravitational dynamics, scalar-field evolution, or modified gravity. The present work instead develops a boundary-governed geometric dynamical system whose evolution is determined entirely by measurable boundary availability Carroll (2004); Mukhanov (2005); Liddle and Lyth (2000). The introduction of the free boundary fraction as the unique geometric state variable therefore represents the principal conceptual contribution of the present work and provides the foundation for the mathematical development presented in the following sections.

The geometric concepts underlying the present formulation draw upon established ideas from Euclidean geometry, topology, differential geometry, and geometric measure theory, while applying them in a cosmological setting distinct from conventional relativistic models Coxeter (1969); Berger (1987); Spivak (1979); Munkres (2000); Mattila (1995); Federer (1969).

# 3 Conceptual Framework and Contributions

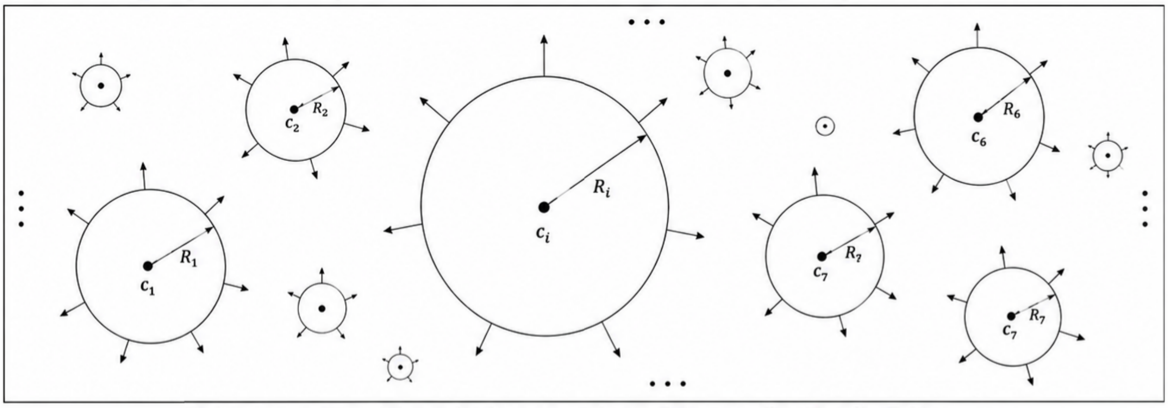
Before developing the mathematical formulation, it is useful to introduce the conceptual picture that motivates the proposed framework. Unlike the standard cosmological paradigm, which considers the evolution of a single expanding universe, the present work investigates an infinite ensemble of independently evolving universes embedded within a common Euclidean space. The geometric language adopted throughout this work follows the classical treatment of Euclidean geometry and differentiable manifolds Berger (1987); Coxeter (1969).

Each universe is represented by an expanding three-dimensional sphere possessing a fixed geometric centre and a time-dependent radius. Expansion is assumed to occur isotropically from the centre of each universe, while interactions between universes arise solely through geometric boundary contact. No assumptions are made regarding matter fields, spacetime curvature, scalar potentials, or gravitational interactions. Consequently, geometry itself constitutes the primary dynamical entity from which the subsequent evolution emerges.

The purpose of this section is to establish the physical intuition underlying the proposed geometric model before introducing its mathematical formalism. Figures 1 and 2 illustrate the conceptual evolution of the system, beginning with an ensemble of independently expanding universes and culminating in the emergence of boundary interactions that regulate subsequent expansion. These figures are intended only as schematic representations of the proposed framework; the quantitative description is developed in the following sections.

## 3.1 An Ensemble of Independently Expanding Universes

**FIGURE 1.** Big Bangs in Euclidean Space (Schematic view of an infinite ensemble of expanding universes)




Each universe originates from a fixed center $c_i$ and expands isotropically with radius $R_i(t)$.

Figure 1: Independent cosmological origins in a Euclidean embedding space. The proposed framework considers an infinite ensemble of independently evolving universes, each represented by an expanding three-dimensional sphere with fixed geometric centre $c_i$ and time-dependent radius $R_i(t)$. Expansion is assumed to occur isotropically from every centre, while no geometric interactions exist during the initial stage of evolution. Differences in subsequent expansion histories arise solely from the spatial arrangement of neighbouring universes within the common embedding space. The figure is schematic and not drawn to scale.

Figure 1 illustrates the fundamental conceptual assumption of the proposed framework. Rather than postulating a single cosmological origin, the model considers an infinite ensemble of universes, each possessing an independent geometric origin represented by a fixed centre $c_i$. Every universe undergoes isotropic radial expansion, beginning from its own initial state and evolving according to its individual expansion history. At this stage, each universe expands independently, without geometric interaction with neighbouring universes.

Because the universes occupy a common Euclidean embedding space, their spatial neighbourhoods need not be identical. Consequently, the subsequent geometric evolution of one universe is generally influenced by the distribution of neighbouring universes rather than by any intrinsic difference in its initial construction. As expansion proceeds, some universes remain relatively isolated for extended periods, whereas others encounter neighbouring expanding boundaries much earlier owing solely to their local geometric environment. This naturally permits heterogeneous expansion histories across the ensemble without introducing additional physical parameters or dynamical fields.

The framework therefore replaces the notion of a unique global inflationary history with a collection of independent boundary-driven expansion histories. Each universe is characterised only by its centre $c_i$, its evolving radius $R_i(t)$, and its surrounding geometric neighbourhood. The representation of universes as geometric balls embedded in Euclidean space follows standard constructions from classical geometry Berger (1987); Coxeter (1969). The diversity of expansion rates emerges collectively from the geometry of the ensemble itself rather than from differences in matter content, vacuum energy, or scalar-field dynamics.

Figure 1 should therefore be interpreted as defining the geometric objects that constitute the model rather than depicting a particular stage of cosmological evolution. The essential consequence of this construction is that independently expanding universes embedded within a common space inevitably approach one another as their radii increase. Once neighbouring boundaries intersect, the expansion of each universe is no longer governed solely by its own geometry but becomes increasingly influenced by collective boundary interactions throughout the ensemble. This transition provides the motivation for introducing the free boundary fraction developed in the following subsection.

## 3.2 Emergence of Boundary Interactions

As the universes introduced in Figure 1 continue to expand, their boundaries inevitably approach those of neighbouring universes embedded within the same Euclidean space. Once the expanding boundaries of two or more universes intersect, the evolution of each universe can no longer be regarded as purely independent. Instead, geometric interactions emerge naturally through local boundary overlap, giving rise to a collective expansion process governed entirely by boundary geometry.

**FIGURE 2.** Boundary Interactions and Free Boundary Fraction for Universe $i$

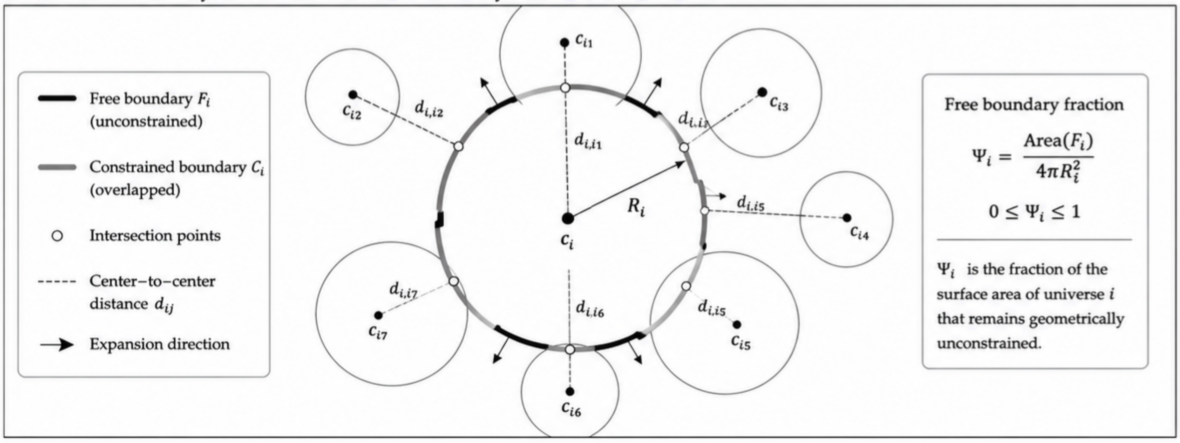



Figure 2: Boundary interactions and the free boundary fraction. As neighbouring universes expand, their boundaries intersect, partitioning the surface of universe $U_i$ into unconstrained (free) boundary segments $F_i$ and constrained (overlapped) boundary segments $C_i$. The free boundary fraction,

$$\Psi_i = \frac{\text{Area}(F_i)}{4\pi R_i^2},$$

quantifies the proportion of the boundary that remains geometrically unconstrained and serves as the sole state variable governing boundary-driven expansion. Increasing overlap reduces $\Psi_i$, thereby producing a self-regulating geometric expansion mechanism. The figure is schematic and not drawn to scale.

Figure 2 illustrates this transition for a representative universe $U_i$. Although the geometric centre $c_i$ remains fixed throughout the evolution, neighbouring universes occupy different locations and therefore intersect the boundary of $U_i$ at distinct positions. These intersections partition the surface of the universe into two complementary regions. Such geometric decompositions are analogous to boundary partitions encountered in geometric measure theory Federer (1969); Morgan (2016).

The first consists of boundary segments that remain completely exposed to the surrounding embedding space and are therefore available for continued unconstrained expansion. The second consists of boundary segments that have become geometrically constrained through overlap with neighbouring universes.

The existence of this decomposition relies on standard results concerning measurable subsets of smooth manifolds and geometric measure theory Taylor (1076).

The central hypothesis of the proposed framework is that only the unconstrained portion of the boundary actively contributes to subsequent radial expansion. Consequently, the instantaneous expansion behaviour of a universe is determined not by its total surface area, but by the fraction of that surface which remains geometrically free. As neighbouring overlaps accumulate, the available free boundary progressively decreases, naturally reducing the effective expansion capacity of the universe without requiring external forces, scalar fields, vacuum energy, or modified gravitational dynamics.

An important consequence of this construction is that the expansion history of a universe becomes an emergent property of its geometric environment. Universes located within relatively sparse neighbourhoods accumulate few boundary intersections and therefore retain a large proportion of free boundary, allowing rapid expansion to persist for longer periods. Conversely, universes embedded within dense neighbourhoods experience more frequent geometric interactions, causing their available free boundary to diminish more rapidly and producing a progressive stagnation of expansion. The model therefore predicts a continuous spectrum of expansion histories arising solely from collective geometric interactions.

The free and constrained boundary regions shown schematically in Figure 2 motivate the introduction of the *free boundary fraction*, which serves as the unique state variable governing the boundary-driven dynamics developed in the mathematical sections that follow. Through this single geometric quantity, local boundary interactions collectively regulate the evolution of each universe while preserving the simplicity of the underlying dynamical framework.

## 3.3 Principal Conceptual Contributions

The proposed framework departs from conventional cosmological models by treating geometry itself as the primary dynamical agent governing cosmic expansion. Conventional descriptions of primordial expansion have overwhelmingly been formulated within the framework of inflationary field dynamics and relativistic cosmology Martin et al. (2014); Bassett et al. (2006). Rather than prescribing expansion through scalar fields, vacuum energy, or modified gravitational dynamics, the present work investigates whether the collective geometry of expanding boundaries alone can generate heterogeneous expansion histories within an ensemble of independently evolving universes. The resulting framework introduces several conceptual contributions that distinguish it from existing approaches.

First, the model replaces the notion of a single cosmological origin with an infinite ensemble of independent cosmological origins embedded within a common Euclidean space. Each universe evolves from its own geometric centre while interacting with neighbouring universes only through local boundary intersections. The framework therefore permits multiple simultaneous expansion histories without requiring a preferred global origin or a unique universal expansion trajectory. Multiple cosmological origins have previously been discussed in the context of eternal inflation and universe nucleation, although through fundamentally different physical mechanisms than those proposed here Linde (1986); Borde et al. (2003).

Second, expansion is formulated as a purely boundary-driven process. The evolution of each universe is governed exclusively by the availability of unconstrained boundary rather than by the dynamics of matter, radiation, scalar fields, or spacetime curvature. This contrasts with conventional inflationary models in which scalar-field evolution governs both the onset and termination of primordial expansion Lyth and Riotto (1999); Martin et al. (2014). As neighbouring universes intersect, the available boundary progressively decreases, producing a self-regulating geometric mechanism in which expansion naturally slows as geometric constraints accumulate.

Third, the framework predicts heterogeneous expansion histories as an emergent consequence of local geometric environments. Universes embedded within sparse neighbourhoods retain a larger fraction of unconstrained boundary and therefore continue expanding for longer periods, whereas universes surrounded by numerous neighbours accumulate boundary constraints more rapidly, leading to progressively stagnated expansion. Diversity in expansion histories therefore arises naturally from collective geometry rather than from differences in intrinsic physical parameters. In standard cosmology, differences in expansion histories are instead associated with gravitational dynamics, matter content, or inflationary initial conditions Ellis et al. (2018); Baumann and Green (2023).

A further conceptual implication is the emergence of evolving networks of boundary intersections. As expansion proceeds, repeated geometric overlaps generate regions of increasingly complex boundary connectivity. Although the present work does not model gravitation, matter, or light propagation, such overlap-rich regions may represent geometrically distinguished structures whose possible relationship to large-scale clustering or lensing-like observational phenomena remains an interesting direction for future investigation. Comparable geometric interpretations have received comparatively little attention within the inflationary literature, where emphasis has generally remained on dynamical field evolution rather than collective boundary interactions Tsujikawa (2003); Barua (2023). These possibilities are presented solely as hypotheses and are not derived within the mathematical framework developed in this paper.

The mathematical development that follows formalizes these ideas through the introduction of the free boundary fraction as the unique geometric state variable governing expansion. Beginning from this single quantity, the subsequent sections derive a closed boundary-driven dynamical system and investigate its mathematical properties independently of any conventional cosmological field equations.

### 3.4 Qualitative Evolution of the Geometric Ensemble

The proposed framework suggests that the long-term evolution of the geometric ensemble is governed entirely by the accumulation of boundary interactions. During the earliest stages of expansion, neighbouring universes are sufficiently separated that most boundaries remain unconstrained. Consequently, the majority of universes initially experience nearly unrestricted radial expansion. As expansion proceeds, however, the probability of boundary contact increases, producing an ever-growing network of geometric intersections throughout the ensemble. The resulting transition from an initially unconstrained regime toward progressively constrained expansion is intended as a qualitative geometric analogue of the inflationary transition rather than a reproduction of conventional exponential inflation Guth (1981); Starobinsky (1980).

Because boundary interactions depend exclusively upon the local geometric neighbourhood of each universe, no two universes are expected to follow identical expansion histories. Universes situated within relatively sparse regions encounter fewer neighbouring boundaries and therefore retain larger unconstrained boundary fractions for longer periods. In contrast, universes embedded within densely populated neighbourhoods accumulate boundary constraints more rapidly, progressively reducing their available free boundary and consequently slowing their expansion. The proposed framework therefore predicts an ensemble exhibiting a continuous spectrum of boundary-driven expansion histories rather than a single universal evolutionary trajectory.

As boundary interactions continue to accumulate, some universes may approach a state in which only a negligible fraction of their boundary remains geometrically unconstrained. Within the present model, such universes are expected to exhibit stagnated expansion, reflecting the progressive exhaustion of available free boundary rather than gravitational collapse or contraction. Conversely, universes experiencing relatively few geometric interactions continue to retain substantial free boundary and therefore sustain comparatively higher expansion rates. Geometric stagnation thus emerges naturally as a consequence of collective boundary saturation.

The repeated accumulation of boundary intersections also gives rise to evolving networks of overlap-rich regions distributed throughout the ensemble. Although the present work does not include matter, gravitation, or light propagation, these geometrically distinguished regions may provide an interesting direction for future investigation. In particular, persistent overlap networks could potentially serve as geometric precursors to large-scale clustering or lensing-like observational phenomena. Such possibilities remain speculative and are not derived within the mathematical framework presented in this paper Feeney et al. (2011); Kleban (2011).

The qualitative behaviour described above motivates the mathematical formulation developed in the following sections. Rather than prescribing the expansion history of each universe through external physical fields or gravitational equations, the proposed theory characterizes expansion using a single geometric quantity that measures the proportion of boundary remaining unconstrained. The subsequent mathematical development demonstrates how this quantity alone is sufficient to generate a self-regulating boundary-driven dynamical system.

## 4 Mathematical Framework

This section develops the mathematical foundation of the proposed model as a purely geometric dynamical system. The framework is constructed from first principles using an infinite family of expanding three-dimensional spheres embedded in a fixed Euclidean space. No assumptions concerning gravitation, matter, energy, pressure, spacetime curvature, or field equations are introduced at this stage. Instead, the evolution of each universe is described exclusively through the geometry of its boundary and its interactions with neighboring boundaries. The resulting formulation establishes a self-contained mathematical theory whose cosmological implications are discussed only in subsequent sections.

The present framework does not reproduce the exponential expansion of conventional inflationary cosmology. Instead, it predicts an initial phase of maximum boundary-driven expansion followed by a self-regulated reduction arising from collective geometric constraints. The correspondence with inflation is therefore qualitative rather than dynamical. The purpose of the present work is to investigate whether characteristic features of the inflationary transition may emerge from boundary geometry alone, independent of scalar-field dynamics.

### 4.1 Geometric Representation of Universes

Let $\mathbb{E}^3$ denote the three-dimensional Euclidean space endowed with the standard Euclidean metric. We consider a countably infinite family of universes represented by closed three-dimensional balls,

$$\mathcal{U} = \{U_i\}_{i=1}^{\infty}, \tag{1}$$

where the $i^{\text{th}}$ universe is defined as

$$U_i(t) = \left\{ x \in \mathbb{E}^3 \,:\, \|x - c_i\| \leq R_i(t) \right\}. \tag{2}$$

Here, $c_i \in \mathbb{E}^3$ denotes the fixed geometric center of the $i^{\text{th}}$ universe, while $R_i(t) \geq 0$ denotes its radius at the evolution parameter $t$. Throughout the mathematical framework, the parameter $t$ serves only to order the geometric evolution of the system and carries no physical interpretation.

The notation and differential-geometric conventions follow standard references do Carmo (1976); Lee (2013).

The boundary of each universe is the corresponding two-dimensional sphere,

$$\partial U_i = \left\{ x \in \mathbb{E}^3 \,:\, \|x - c_i\| = R_i(t) \right\}, \tag{3}$$

whose surface area and enclosed volume are respectively given by

$$S_i = 4\pi R_i^2, \tag{4}$$

and

$$V_i = \frac{4}{3}\pi R_i^3. \tag{5}$$

The collection $\mathcal{U}$ constitutes the complete geometric domain of the proposed framework. Each universe is treated solely as a geometric object characterized by its center, radius, boundary, surface area, and enclosed volume. At this stage, no assumptions are made regarding the physical nature of the universes or the mechanism responsible for their expansion. The subsequent development derives the dynamics entirely from the geometric interactions of their evolving boundaries.

### 4.2 Geometric Axioms

The proposed framework is founded upon a set of geometric axioms that govern the evolution of the universe ensemble. These axioms are independent of any physical interpretation and serve as the mathematical basis for all subsequent derivations.

1. **Axiom 1 (Infinite Universe Ensemble).** The geometric domain consists of a countably infinite collection of universes,
$$\mathcal{U} = \{U_i\}_{i=1}^{\infty}, \tag{6}$$
each represented as a closed three-dimensional ball embedded in $\mathbb{E}^3$.

2. **Axiom 2 (Fixed Geometric Centers).** Every universe possesses a fixed geometric center,
$$c_i = \text{constant}, \tag{7}$$
such that the pairwise distances
$$d_{ij} = \|c_i - c_j\| \tag{8}$$
remain invariant throughout the evolution.

3. **Axiom 3 (Differentiable Expansion).** The radius of every universe is a continuously differentiable function of the evolution parameter,
$$R_i \in C^1, \tag{9}$$
ensuring that the geometric evolution is continuous.
4. **Axiom 4 (Boundary-Limited Expansion).** Expansion occurs exclusively through the boundary of each universe. Interior points remain geometrically unchanged, while the boundary evolves according to the availability of unconstrained surface area.
5. **Axiom 5 (Geometric Interaction).** Interactions between universes arise solely through boundary contact. No long-range interactions or external forces are assumed. Consequently, all dynamical behavior emerges from local geometric boundary constraints.

These five axioms completely specify the mathematical setting of the proposed theory. All subsequent definitions, theorems, and dynamical equations are derived exclusively from these geometric assumptions without introducing additional physical postulates.

### 4.3 Boundary Contact Conditions

The geometric interaction between universes is determined entirely by the relative positions of their boundaries. Since the centers remain fixed by Axiom 2, the spatial relationship between any two universes is governed solely by their radii and the constant Euclidean distance separating their centers.

For any pair of universes $U_i$ and $U_j$, let

$$d_{ij} = \|c_i - c_j\| \tag{10}$$

denote the fixed Euclidean distance between their centers.

The following mutually exclusive geometric configurations are possible.

1. **Disjoint Universes.** The boundaries do not intersect when
$$R_i + R_j < d_{ij}. \tag{11}$$
2. **Boundary Contact.** The two boundaries are tangent when
$$R_i + R_j = d_{ij}. \tag{12}$$
This configuration represents the onset of geometric interaction.
3. **Boundary Overlap.** The boundaries intersect whenever
$$R_i + R_j > d_{ij}. \tag{13}$$
The intersection defines a common constrained region shared by both universes.

Since the centers remain fixed throughout the evolution, changes in the contact configuration arise exclusively from the expansion of the radii. Consequently, every geometric interaction within the ensemble is completely determined by the evolving boundary geometry.

The occurrence of boundary overlap partitions the surface of each universe into regions that remain geometrically unconstrained and regions that participate in one or more intersections. This partition forms the basis for the boundary decomposition introduced in the following subsection.

## 4.4 Boundary Decomposition

Whenever the boundary of a universe intersects the boundary of one or more neighboring universes, its surface naturally separates into geometrically distinct regions. This induces a decomposition of the boundary into unconstrained and constrained components.

Let $F_i \subseteq \partial U_i$ denote the *free boundary*, consisting of all boundary points that do not belong to the intersection with any neighboring universe. Likewise, let $C_i \subseteq \partial U_i$ denote the *constrained boundary*, comprising all boundary points that participate in one or more boundary intersections.

Accordingly, the boundary of the $i^{\text{th}}$ universe admits the disjoint decomposition

$$\partial U_i = F_i \,\dot{\cup}\, C_i, \tag{14}$$

where $\dot{\cup}$ denotes a disjoint union. Consequently,

$$F_i \cap C_i = \varnothing. \tag{15}$$

Since every boundary point belongs uniquely to either the free or the constrained region, the surface measure satisfies

$$\text{Area}(F_i) + \text{Area}(C_i) = \text{Area}(\partial U_i) = 4\pi R_i^2. \tag{16}$$

Equation (16) expresses the conservation of boundary measure under the geometric partition. Although the relative sizes of $F_i$ and $C_i$ evolve as additional boundary intersections occur, their combined surface area always equals the total surface area of the spherical boundary.

The existence of this decomposition relies on standard results concerning measurable subsets of smooth manifolds and geometric measure theory Federer (1969); Morgan (2016).

The free boundary represents the portion of the surface that remains geometrically available for unconstrained expansion, whereas the constrained boundary corresponds to regions whose outward growth is restricted by boundary overlap with neighboring universes. This geometric decomposition provides the fundamental quantity from which the expansion dynamics of the proposed framework are subsequently derived.

## 4.5 Free Boundary Fraction

The geometric decomposition established in the previous subsection naturally gives rise to a dimensionless measure of the boundary available for unconstrained expansion. We therefore define the *free boundary fraction* of the $i^{\text{th}}$ universe as

$$\Psi_i = \frac{\text{Area}(F_i)}{\text{Area}(\partial U_i)} = \frac{\text{Area}(F_i)}{4\pi R_i^2}. \tag{17}$$

Since the free boundary is a subset of the total boundary,

$$0 \le \text{Area}(F_i) \le 4\pi R_i^2, \tag{18}$$

it immediately follows that

$$0 \le \Psi_i \le 1. \tag{19}$$

The limiting values admit a natural geometric interpretation. When

$$\Psi_i = 1, \tag{20}$$

the entire boundary is free, implying that the universe is geometrically unconstrained by its neighbors. Conversely,

$$\Psi_i = 0, \tag{21}$$

indicates that the entire boundary is constrained through boundary overlap.

More generally, intermediate values,

$$0 < \Psi_i < 1, \tag{22}$$

represent partial geometric constraint, where only a fraction of the boundary remains available for unconstrained expansion.

The free boundary fraction is the sole dimensionless state variable introduced in the proposed framework. It encapsulates the cumulative geometric influence of all neighboring universes and serves as the fundamental quantity governing the expansion dynamics developed in the following subsection.

The mathematical treatment of measurable boundary subsets and compact geometric domains follows standard principles of analysis and measure theory Rudin (1976); Evans and Gariepy (2015).

## 4.6 Properties of the Free Boundary Functional

The free boundary fraction introduced in Equation (19) serves as the unique state variable governing the proposed geometric dynamics. Although no closed-form expression for $\Psi_i$ exists for an arbitrary infinite ensemble of intersecting universes, several fundamental mathematical properties follow directly from the geometric construction. These properties establish that $\Psi_i$ is a well-defined geometric functional and justify its use in the governing expansion law.

**Theorem 1** (Existence)**.** *For every universe $U_i$, the free boundary fraction*

$$\Psi_i = \frac{\text{Area}(F_i)}{4\pi R_i^2}$$

*is well defined.*

*Proof.* Each universe is represented by a closed ball in $\mathbb{E}^3$ with boundary $\partial U_i$, which is a compact two-dimensional sphere. Since the constrained boundary is constructed from finite or countable unions of boundary intersections, the free boundary

$$F_i = \partial U_i \setminus C_i$$

is measurable with respect to the surface measure on $\partial U_i$. Consequently, $\text{Area}(F_i)$ exists, and because $R_i > 0$, the denominator $4\pi R_i^2$ is strictly positive. Therefore, $\Psi_i$ is well defined. □

**Theorem 2** (Bounds)**.** *For every universe,*

$$0 \leq \Psi_i \leq 1.$$

*Proof.* Since

$$F_i \subseteq \partial U_i,$$

it follows immediately that

$$0 \leq \text{Area}(F_i) \leq \text{Area}(\partial U_i) = 4\pi R_i^2.$$

Dividing by $4\pi R_i^2 > 0$ yields

$$0 \leq \Psi_i \leq 1.$$

□

**Theorem 3** (Monotonicity Under Increasing Boundary Constraints)**.** *Suppose that during the geometric evolution no previously constrained boundary becomes free, so that*

$$C_i(t_1) \subseteq C_i(t_2), \qquad t_2 > t_1.$$

*Then the free boundary fraction satisfies*

$$\Psi_i(t_2) \leq \Psi_i(t_1).$$

*Proof.* Since

$$F_i(t) = \partial U_i \setminus C_i(t),$$

the assumption

$$C_i(t_1) \subseteq C_i(t_2)$$

implies

$$F_i(t_2) \subseteq F_i(t_1).$$

Therefore,

$$\mathrm{Area}(F_i(t_2)) \leq \mathrm{Area}(F_i(t_1)).$$

Because the denominator in the definition of $\Psi_i$ is positive,

$$\Psi_i(t_2) = \frac{\mathrm{Area}(F_i(t_2))}{4\pi R_i^2} \leq \frac{\mathrm{Area}(F_i(t_1))}{4\pi R_i^2} = \Psi_i(t_1).$$

□

Under the assumptions of the preceding theorem, the governing expansion law

$$\frac{dR_i}{dt} = v_0 \Psi_i$$

implies

$$\left(\frac{dR_i}{dt}\right)_{t_2} \leq \left(\frac{dR_i}{dt}\right)_{t_1}, \qquad t_2 > t_1.$$

Hence, increasing geometric constraint can only reduce the instantaneous radial expansion rate.

**Theorem 4** (Limiting Configurations)**.** *The free boundary fraction admits the limiting values*

$$\Psi_i = 1$$

*if and only if the boundary is completely unconstrained, and*

$$\Psi_i = 0$$

*if and only if the entire boundary is constrained.*

*Proof.* If the boundary is entirely free,

$$F_i = \partial U_i,$$

then

$$\mathrm{Area}(F_i) = 4\pi R_i^2,$$

so that

$$\Psi_i = 1.$$

Conversely, if the entire boundary is constrained,

$$F_i = \varnothing,$$

then

$$\mathrm{Area}(F_i) = 0,$$

which yields

$$\Psi_i = 0.$$

These are precisely the two extreme geometric configurations admitted by the definition of the free boundary fraction. □

## 4.7 Expansion Dynamics

Having established the geometric representation of universes and the free boundary fraction, we now formulate the governing law of boundary evolution. The proposed framework postulates that the instantaneous radial expansion of a universe is proportional to the fraction of its boundary that remains geometrically unconstrained. Accordingly, the radius of the $i^{\text{th}}$ universe evolves according to

$$\frac{dR_i}{dt} = v_0 \, \Psi_i, \tag{23}$$

where $v_0 > 0$ is the intrinsic radial expansion constant and $\Psi_i$ is the free boundary fraction defined in Equation (17).

Equation (23) constitutes the sole constitutive law of the proposed framework. Since $0 \leq \Psi_i \leq 1$, it immediately follows that

$$0 \leq \frac{dR_i}{dt} \leq v_0. \tag{24}$$

Thus, the intrinsic expansion constant represents the maximum attainable radial expansion rate, achieved only when the boundary is completely unconstrained.

The governing equation further determines the evolution of the boundary surface area. Since

$$S_i = 4\pi R_i^2, \tag{25}$$

differentiation with respect to the evolution parameter yields

$$\frac{dS_i}{dt} = 8\pi R_i \frac{dR_i}{dt} = 8\pi R_i v_0 \Psi_i. \tag{26}$$

Similarly, the enclosed volume,

$$V_i = \frac{4}{3}\pi R_i^3, \tag{27}$$

satisfies

$$\frac{dV_i}{dt} = 4\pi R_i^2 \frac{dR_i}{dt} = 4\pi R_i^2 v_0 \Psi_i. \tag{28}$$

Dividing Equation (28) by $V_i$ gives the relative volumetric growth rate,

$$\frac{1}{V_i}\frac{dV_i}{dt} = \frac{3 v_0 \Psi_i}{R_i}. \tag{29}$$

Equations (26)–(29) are direct mathematical consequences of the governing expansion law and require no additional assumptions. The evolution of every geometric quantity within the proposed framework is therefore determined entirely by the free boundary fraction.

## 4.8 Closed Geometric Dynamical System

The mathematical framework developed in the preceding subsections is completely determined by the geometric evolution of the free boundary fraction. Since the constrained boundary arises solely through boundary intersections within the universe ensemble, the free boundary fraction is itself a geometric function of the evolving configuration,

$$\Psi_i = \Psi_i(\mathcal{U}), \tag{30}$$

where $\mathcal{U}$ denotes the complete collection of expanding universes. Consequently, the evolution of each universe depends only on the instantaneous geometric configuration of the ensemble.

Substituting Equation (30) into the governing expansion law (Equation 23) yields

$$\frac{dR_i}{dt} = v_0\, \Psi_i(\mathcal{U}). \tag{31}$$

Equation (31) represents a self-contained geometric dynamical system. The evolution of each universe is governed exclusively by its boundary geometry and the cumulative geometric influence of neighboring universes. No external forcing terms, source functions, field variables, or non-geometric interactions are introduced.

The proposed framework therefore reduces the dynamics of the entire universe ensemble to the evolution of a single dimensionless geometric quantity, namely the free boundary fraction. As the configuration of the ensemble changes through boundary interactions, the available free boundary evolves accordingly, thereby regulating the instantaneous radial expansion of every universe.

The mathematical framework is therefore fully specified by the following components:

1. the geometric representation of universes as expanding closed balls in $\mathbb{E}^3$;
2. the decomposition of each boundary into free and constrained regions;
3. the free boundary fraction defined by Equation (17); and
4. the governing expansion law given by Equation (23).

Together, these elements constitute a complete geometric theory of boundary-driven expansion. All subsequent physical interpretations are developed from this mathematical framework without modification of its underlying geometric structure.

# 5 Geometric Consequences

The mathematical framework established in Section 4 permits a number of immediate consequences that arise directly from the governing equations. These results are derived solely from the geometry of the expanding universe ensemble and therefore require no additional physical assumptions. In particular, the evolution of every universe is governed entirely by the availability of its free boundary, making the geometric configuration of the ensemble the unique determinant of its expansion history. The following subsections examine the principal consequences of this boundary-driven dynamics before considering their cosmological interpretation.

## 5.1 Maximum Expansion Phase

The governing expansion law introduced in Equation (23),

$$\frac{dR_i}{dt} = v_0 \Psi_i, \tag{32}$$

together with the geometric bound

$$0 \leq \Psi_i \leq 1, \tag{33}$$

immediately implies

$$0 \leq \frac{dR_i}{dt} \leq v_0. \tag{34}$$

Hence, the maximum attainable radial expansion rate is achieved if and only if

$$\Psi_i = 1. \tag{35}$$

This condition corresponds to the absence of geometric constraints, whereby the entire boundary of the universe remains available for unconstrained expansion. Under these circumstances,

$$\frac{dR_i}{dt} = v_0, \tag{36}$$

which integrates to

$$R_i(t) = R_i(0) + v_0 t. \tag{37}$$

Equation (37) represents the unique unconstrained solution of the proposed geometric framework. It follows that every universe initially evolves at its intrinsic maximum radial expansion rate until the development of boundary interactions reduces the available free boundary. Consequently, unconstrained expansion is not an additional assumption of the model but a direct mathematical consequence of the absence of geometric constraints.

## 5.2 Emergence of Geometric Constraints

As the universe ensemble evolves, expanding boundaries progressively encounter neighboring boundaries, giving rise to constrained surface regions as defined in Section 4.4. Consequently, the free boundary fraction is no longer constant but evolves as a function of the instantaneous geometric configuration of the ensemble.

Since the governing expansion law is

$$\frac{dR_i}{dt} = v_0 \Psi_i, \tag{38}$$

any reduction in the free boundary fraction necessarily produces a corresponding reduction in the instantaneous radial expansion rate. Thus, for two successive geometric configurations satisfying

$$\Psi_i^{(2)} < \Psi_i^{(1)}, \tag{39}$$

it immediately follows that

$$\left(\frac{dR_i}{dt}\right)^{(2)} < \left(\frac{dR_i}{dt}\right)^{(1)}. \tag{40}$$

This result demonstrates that the expansion rate decreases as the available free boundary diminishes. No additional damping mechanism, external interaction, or phenomenological correction is required. The reduction in expansion arises solely from the geometric redistribution of the boundary into free and constrained regions.

It is important to note that the constrained boundary is not generated by a single neighboring universe. Rather, it represents the cumulative effect of boundary intersections with the surrounding universe ensemble. Consequently, the free boundary fraction should be regarded as a collective geometric property,

$$\Psi_i = \Psi_i(\mathcal{U}), \tag{41}$$

where $\mathcal{U}$ denotes the complete ensemble of expanding universes. The instantaneous expansion rate of each universe is therefore determined by the global geometric configuration of the ensemble rather than by isolated pairwise interactions.

Accordingly, the proposed framework predicts that the transition from unconstrained to constrained expansion is an emergent geometric phenomenon arising from the collective evolution of the boundary network. This transition follows directly from the mathematical structure of the model and requires no additional postulates beyond those established in Section 4.

### 5.3 Geometric Interpretation of the Inflationary Phase

The mathematical framework developed in the preceding sections admits a natural geometric interpretation of an early phase of unconstrained expansion. From Equation (23), the condition

$$\Psi_i = 1 \tag{42}$$

implies

$$\frac{dR_i}{dt} = v_0, \tag{43}$$

indicating that the universe expands at its intrinsic maximum radial rate. This regime persists only while the boundary remains entirely free of geometric constraints.

As the expanding universe becomes embedded within the evolving ensemble, boundary intersections progressively reduce the available free boundary. Consequently,

$$\Psi_i < 1, \tag{44}$$

and the governing expansion law becomes

$$\frac{dR_i}{dt} = v_0 \Psi_i < v_0. \tag{45}$$

The reduction in the expansion rate therefore follows directly from the emergence of collective boundary constraints and requires no additional geometric assumptions.

Within the proposed framework, this transition provides a geometric correspondence to the transition from an initial phase of rapid expansion to a subsequent phase of slower evolution. Accordingly, the unconstrained regime ($\Psi_i = 1$) may be interpreted as the geometric analogue of an inflationary epoch, while the progressive reduction of the free boundary offers a purely geometric mechanism for its termination.

It is important to emphasize that this correspondence is interpretative rather than axiomatic. The mathematical framework itself is entirely independent of cosmological assumptions. Instead, the inflationary interpretation arises as a consequence of comparing the geometric behavior predicted by the model with the qualitative features of the conventional cosmological expansion history.

### 5.4 Characteristic Transition Scale

The proposed framework predicts that the transition from unconstrained to constrained expansion occurs when the cumulative boundary interactions within the universe ensemble become sufficiently significant to reduce the free boundary fraction below unity. Although the mathematical framework does not prescribe a numerical value for this transition radius, it establishes the existence of a finite geometric scale at which the expansion law changes from

$$\frac{dR_i}{dt} = v_0, \tag{46}$$

to

$$\frac{dR_i}{dt} = v_0 \Psi_i, \qquad 0 < \Psi_i < 1. \tag{47}$$

This transition is governed entirely by the evolving geometry of the ensemble and therefore represents an intrinsic property of the proposed model rather than an externally imposed condition.

A quantitative comparison with conventional inflationary cosmology is informative. Standard inflationary models predict that the observable universe undergoes approximately sixty $e$-folds of expansion, corresponding to a physical radius of the order of $10^{-1}$–$10^{0}$ m at the conclusion of the inflationary epoch. Typical estimates of the inflationary duration and characteristic physical scales may be found in Mukhanov (2005); Liddle and Lyth (2000). The finite transition radius predicted by the present framework is therefore naturally comparable to this characteristic scale.

This agreement should be regarded as a numerical correspondence rather than a derivation. The mathematical framework neither assumes nor requires the conventional inflationary scenario. Instead, it independently predicts the existence of a finite geometric transition between unconstrained and constrained expansion. The proximity of this transition to the characteristic inflationary length scale provides a potential point of contact between the proposed geometric framework and established cosmological models.

Consequently, the theory predicts that the termination of the initial maximum-expansion phase is governed by the emergence of collective geometric constraints rather than by the introduction of additional physical fields or phenomenological mechanisms. The observational implications of this correspondence are discussed in the following section.

## 5.5 Self-Regulation of Expansion

A distinguishing feature of the proposed framework is that the evolution of each universe is governed entirely by its own geometric state. From the governing expansion law,

$$\frac{dR_i}{dt} = v_0 \Psi_i, \tag{48}$$

the instantaneous expansion rate is determined solely by the free boundary fraction, which is itself a consequence of the evolving boundary geometry.

As the universe expands, additional boundary intersections naturally increase the constrained boundary region, thereby reducing the free boundary fraction. Consequently, the governing equation automatically decreases the radial expansion rate according to

$$\Psi_i \downarrow \implies \frac{dR_i}{dt} \downarrow . \tag{49}$$

Conversely, if the geometric configuration admits a larger free boundary, the expansion rate increases correspondingly. The governing dynamics therefore establish a continuous feedback between the evolving geometry of the boundary and the instantaneous rate of expansion.

This behaviour is entirely intrinsic to the mathematical framework. No auxiliary damping terms, correction factors, external fields, phenomenological potentials, or additional evolution equations are required to regulate the expansion. Instead, the geometry of the universe ensemble provides its own mechanism for controlling the evolution through the availability of unconstrained boundary.

Accordingly, the proposed model exhibits a self-regulating expansion process in which the expansion rate continuously adapts to the collective geometric configuration of the ensemble. This self-regulation arises directly from the governing equations and represents a fundamental consequence of the boundary-driven dynamics.

## 5.6 Testable Predictions

A fundamental requirement of any theoretical framework is that it should admit observational or experimental verification. Although the proposed model is formulated as a purely geometric theory, the preceding analysis gives rise to several qualitative predictions that distinguish it from conventional cosmological models.

First, the framework predicts that the transition from unconstrained to constrained expansion occurs at a finite geometric scale determined by the collective evolution of the universe ensemble. This transition is not imposed externally but emerges naturally from the reduction of the free boundary fraction.

Second, the proposed theory attributes the reduction of the expansion rate to cumulative geometric boundary interactions rather than to an externally introduced inflationary field or an ad hoc termination mechanism. Consequently, any observational signatures associated with the transition should ultimately reflect the geometry of the evolving boundary network.

Third, because the free boundary fraction depends on the global geometric configuration of the ensemble,

$$\Psi_i = \Psi_i(\mathcal{U}), \tag{50}$$

the expansion history of an individual universe is predicted to be an emergent collective property rather than the consequence of isolated pairwise interactions.

These predictions follow directly from the mathematical framework developed in Section 4. Their quantitative assessment requires comparison with cosmological observations and numerical simulations, which lie beyond the scope of the present work. Nevertheless, the framework provides a well-defined set of geometric predictions that are, in principle, capable of distinguishing the proposed model from conventional descriptions of the early universe.

# 6 Cosmological Interpretation and Discussion

The preceding sections developed a purely geometric framework for the evolution of an infinite ensemble of expanding universes without invoking gravitation, spacetime curvature, matter fields, or inflationary potentials. The present section considers the possible cosmological interpretation of the resulting mathematical structure. The objective is not to replace established cosmological models, but to examine whether the geometric consequences derived from the proposed framework exhibit qualitative correspondences with known features of the early universe. Throughout this discussion, the distinction between mathematical derivation and physical interpretation is maintained. Accordingly, the following observations should be regarded as interpretative consequences of the geometric model rather than assumptions upon which the framework is constructed.

## 6.1 Relation to Standard Inflation

One of the most striking consequences of the proposed framework is the existence of an initial phase of unconstrained expansion. From the governing equation,

$$\frac{dR_i}{dt} = v_0 \Psi_i, \tag{51}$$

the condition

$$\Psi_i = 1 \tag{52}$$

implies that the universe expands at its intrinsic maximum radial rate. This behaviour continues until collective boundary interactions reduce the available free boundary.

Standard inflationary cosmology similarly postulates an early epoch of extremely rapid expansion followed by a transition to a slower evolutionary phase. Reviews of the conventional inflationary mechanism are provided by Mukhanov (2005); Bassett et al. (2006). Within the proposed framework, this qualitative behaviour emerges naturally from the geometry of the universe ensemble. The unconstrained regime corresponds to complete boundary availability, whereas the subsequent reduction in the free boundary fraction provides an intrinsic geometric mechanism for decreasing the expansion rate.

An important distinction, however, is that the present framework does not introduce an inflaton field, a scalar potential, or an externally prescribed reheating mechanism. Instead, the transition from maximum expansion to reduced expansion is interpreted as a consequence of the collective evolution of boundary geometry. In this sense, the proposed model offers an alternative geometric interpretation of the inflationary transition while remaining agnostic regarding the detailed microphysical processes responsible for the early universe.

### 6.2 Inflation Termination Through Collective Geometry

In conventional inflationary cosmology, the transition from the inflationary epoch to the subsequent expansion history is generally attributed to the evolution of an inflationary scalar field and the dynamics of its associated potential. This mechanism differs fundamentally from the slow-roll paradigm developed in conventional inflationary cosmology Lyth and Riotto (1999); Martin et al. (2014). By contrast, the proposed framework attributes this transition entirely to the collective geometry of the expanding universe ensemble.

Within the mathematical framework developed in Section 4, the expansion of an individual universe is governed by

$$\frac{dR_i}{dt} = v_0 \Psi_i, \tag{53}$$

where the free boundary fraction $\Psi_i$ measures the proportion of the boundary that remains geometrically unconstrained. During the initial stage of evolution,

$$\Psi_i = 1, \tag{54}$$

and the universe expands at its intrinsic maximum radial rate. As the surrounding universe ensemble evolves, boundary intersections progressively reduce the available free boundary, leading to

$$0 < \Psi_i < 1. \tag{55}$$

The resulting decrease in the expansion rate follows directly from the governing equation without requiring additional dynamical variables or external termination mechanisms.

A notable feature of the proposed framework is that this reduction does not arise from the interaction between a single pair of universes. Instead, the free boundary fraction is determined by the cumulative geometric influence of the entire ensemble,

$$\Psi_i = \Psi_i(\mathcal{U}), \tag{56}$$

where $\mathcal{U}$ denotes the complete collection of expanding universes. The transition from unconstrained to constrained expansion is therefore an emergent collective phenomenon resulting from the evolving topology of the boundary network.

This interpretation provides a purely geometric mechanism for terminating the initial maximum-expansion phase. Rather than introducing additional physical entities to regulate the expansion, the proposed model attributes the transition to the progressive reduction of geometrically available boundary. The qualitative correspondence between this transition and the conclusion of the inflationary epoch suggests that collective boundary geometry may provide an alternative mathematical description of the early expansion history of the universe.

It should be emphasized that the present work does not claim to replace the standard inflationary paradigm. Instead, it proposes that the observed transition from rapid to slower expansion may admit an equivalent geometric interpretation based solely on boundary interactions within an infinite ensemble of expanding universes. Such an interpretation provides a testable alternative whose quantitative implications remain to be explored in future investigations.

## 6.3 Comparison with Conventional Cosmology

The proposed framework shares several qualitative features with the conventional cosmological description while differing fundamentally in its underlying mathematical construction. Both approaches predict an initial phase of rapid expansion followed by a transition to a slower expansion history. However, the mechanisms responsible for these behaviours are substantially different.

In the standard cosmological paradigm, the evolution of the universe is described by the Einstein field equations together with assumptions regarding the matter-energy content of the universe. The early accelerated expansion is commonly attributed to the dynamics of an inflationary scalar field, while the subsequent evolution is governed by the gravitational interaction between radiation, matter, and dark energy. Classical treatments of relativistic cosmology and spacetime geometry are given by Hawking and Ellis (1973); Wald (1984); Misner et al. (1973).

The present framework adopts a fundamentally different perspective. The mathematical model developed in Section 4 does not invoke spacetime curvature, gravitational field equations, matter fields, or inflationary potentials. Instead, the evolution of each universe is determined entirely by the geometry of its expanding boundary and the collective boundary interactions arising within an infinite ensemble of universes. Consequently, the reduction in the expansion rate emerges as an intrinsic geometric property rather than as the consequence of an externally introduced physical mechanism.

It is important to emphasize that the proposed model is not intended to invalidate the standard cosmological framework. Rather, it offers an alternative geometric interpretation of the early expansion history that reproduces several qualitative features through an entirely different mathematical formulation. Whether this interpretation can also account for quantitative cosmological observations remains an open question requiring further theoretical development and observational investigation.

From this perspective, the proposed framework should be regarded as complementary to existing cosmological theories. Its principal contribution is the introduction of a boundary-driven geometric mechanism capable of generating an initial maximum-expansion phase, a subsequent self-regulated reduction in expansion, and a finite transition scale without introducing additional dynamical fields. Future work will determine whether these geometric principles can be integrated with, or provide an alternative foundation for, conventional cosmological descriptions.

## 6.4 Limitations and Future Directions

The present work establishes a geometric framework for the evolution of an infinite ensemble of expanding universes and demonstrates that several qualitative features commonly associated with the early universe can arise from boundary-driven dynamics. Nevertheless, the proposed model represents an initial mathematical formulation whose scope is intentionally restricted.

The framework has been developed independently of General Relativity and therefore does not presently derive the Einstein field equations, the Friedmann equations, or the standard cosmological evolution equations. Likewise, the present formulation does not explicitly incorporate matter, radiation, dark matter, dark energy, or quantum gravitational effects. These omissions are deliberate, allowing the geometric consequences of boundary interactions to be investigated in isolation before introducing additional physical structure. The present formulation should therefore be regarded as a geometric pre-relativistic model whose future extension to relativistic spacetime may naturally draw upon established treatments of differential geometry and relativistic gravitation O'Neill (1983); Carroll (2004); Schutz (2009). Potential future integration with relativistic cosmology would naturally build upon the mathematical foundations established in Hawking and Ellis (1973); Wald (1984).

Furthermore, the present study focuses on establishing the internal mathematical consistency of the proposed framework rather than providing a complete cosmological model. This philosophy parallels the development of many foundational mathematical models, where internal consistency is established prior to introducing additional physical structure or observational constraints Nakahara (2003); Spivak (1979). Although qualitative correspondences with the inflationary epoch have been identified, a quantitative comparison with cosmological observations—including the cosmic microwave background, large-scale structure, primordial nucleosynthesis, and other precision measurements—remains the subject of future investigation. Modern observational cosmology provides numerous measurable quantities against which future extensions of the present framework could ultimately be evaluated Dodelson and Schmidt (2020); Weinberg (2008).

Several directions for future research naturally emerge from the present work. These include deriving observational predictions from the boundary-driven expansion law, determining the functional dependence of the free boundary fraction on the evolving universe ensemble, developing numerical simulations of collective boundary evolution, and investigating possible connections between the proposed geometric framework and conventional relativistic cosmology. Such simulations would naturally benefit from established techniques in computational and metric geometry for analysing evolving geometric configurations Burago et al. (2001); Thurston (1997).

Despite these limitations, the proposed framework introduces a mathematically self-contained description of boundary-driven expansion that differs fundamentally from existing cosmological formulations. Whether this geometric perspective represents a complementary interpretation of known cosmological phenomena or the foundation of a broader theoretical framework remains an open question for future research.

# 7 Conclusion

This paper has presented a purely geometric framework for describing the evolution of an infinite ensemble of expanding universes. Unlike conventional cosmological models, the proposed theory is constructed entirely from the geometry of expanding spherical boundaries embedded in a fixed Euclidean space, without invoking gravitation, spacetime curvature, matter fields, inflationary scalar potentials, or other physical postulates. The resulting mathematical formulation is governed by a single boundary-driven expansion law in which the instantaneous radial expansion depends exclusively on the fraction of unconstrained boundary available to each universe. A fundamental contribution of the proposed framework is the introduction of the free boundary fraction as the unique geometric state variable governing expansion. From this concept, the model naturally predicts an initial phase of maximum unconstrained expansion, the progressive emergence of collective boundary constraints, and a self-regulating reduction in the expansion rate. These consequences arise directly from the geometry of the evolving universe ensemble and require no auxiliary damping mechanisms or externally prescribed transition conditions.

The framework further admits a natural qualitative correspondence with the inflationary phase of conventional cosmology. Within the proposed interpretation, the initial unconstrained regime provides a geometric analogue of rapid primordial expansion, while the subsequent reduction of the free boundary offers an intrinsic geometric mechanism for its termination. Although this correspondence remains interpretative, it demonstrates that several characteristic features of the early universe may emerge from boundary geometry alone. The present work is intended as a foundational mathematical formulation rather than a complete cosmological theory. This perspective is consistent with the broader scientific practice of developing mathematically self-contained frameworks before investigating their physical realization and observational consequences Penrose (2004); Nakahara (2003). Future investigations should focus on deriving quantitative observational predictions, developing numerical simulations of collective boundary evolution, establishing possible connections with relativistic cosmology, and assessing the compatibility of the framework with precision cosmological observations. Such studies will determine whether boundary-driven geometry can serve as a complementary description of known cosmological phenomena or provide the basis for a broader geometric theory of cosmic evolution.

In summary, the proposed model demonstrates that the collective geometry of an infinite ensemble of expanding universes is sufficient to generate a self-contained boundary-driven dynamical system. By reducing cosmological evolution to the evolution of boundary geometry, the framework offers a novel mathematical perspective on the early expansion history of the universe and provides a foundation for future theoretical and observational investigation. Whether this boundary-driven geometric perspective represents a complementary interpretation of cosmic evolution or the foundation of a broader cosmological framework remains an open question, inviting further mathematical development and observational scrutiny.

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The author declares that there are no competing interests, financial or non-financial, that could have influenced the work reported in this manuscript.

## Author Contributions

The author conceived the study, developed the theoretical framework, performed the mathematical analysis, prepared the figures, wrote the manuscript, and approved the final version for publication.

## Data Availability

No datasets were generated or analysed during the current study. The manuscript presents a theoretical mathematical framework derived from first principles.

## Code Availability

No custom software or computational code was required to derive the theoretical results presented in this work.

## Ethics Approval

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## Consent to Participate

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## References


Aguirre, A. and Johnson, M. C. (2007). Towards observable signatures of other bubble universes. ii. exact solutions for thin-wall bubble collisions. *Physical Review D*, 76(2):023509.

Albrecht, A. and Steinhardt, P. J. (1982). Cosmology for grand unified theories with radiatively induced symmetry breaking. *Physical Review Letters*, 48(17):1220.

Barua, S. M. (2023). Proposed physical mechanism that gives rise to cosmic inflation. *Scientific Reports*, 13(1):21815.

Bassett, B. A., Tsujikawa, S., and Wands, D. (2006). Inflation dynamics and reheating. *Reviews of Modern Physics*, 78:537–589.

Baumann, D. and Green, D. (2023). Inflation (2025). *arXiv preprint arXiv:2312.13238.* Chapter 23 of The Review of Particle Physics 2026.

Berger, M. (1987). *Geometry I.* Springer.

Borde, A., Guth, A. H., and Vilenkin, A. (2003). Inflationary spacetimes are incomplete in past directions. *Physical Review Letters*, 90(15):151301.

Borde, A. and Vilenkin, A. (1994). Eternal inflation and the initial singularity. *Physical Review Letters*, 72(21):3305–3309.

Burago, D., Burago, Y., and Ivanov, S. (2001). *A Course in Metric Geometry.* American Mathematical Society.

Carroll, S. M. (2004). *Spacetime and Geometry: An Introduction to General Relativity.* Addison-Wesley.

Coxeter, H. S. M. (1969). *Introduction to Geometry.* Wiley.

do Carmo, M. P. (1976). *Differential Geometry of Curves and Surfaces.* Prentice Hall.

Dodelson, S. and Schmidt, F. (2020). *Modern Cosmology.* Academic Press, 2 edition.

Ellis, J., Garcia, M. A. G., Nagata, N., Nanopoulos, D. V., and Olive, K. A. (2018). Inflationary cosmology: From theory to observations. *arXiv preprint arXiv:1810.09934.*

Evans, L. C. and Gariepy, R. F. (2015). *Measure Theory and Fine Properties of Functions.* CRC Press, 2 edition.

Federer, H. (1969). *Geometric Measure Theory.* Springer.

Feeney, S. M., Johnson, M. C., Mortlock, D. J., and Peiris, H. V. (2011). First observational tests of eternal inflation. *Physical Review Letters*, 107(7):071301.

Finelli, F., Bucher, M., Achúcarro, A., Ballardini, M., Bartolo, N., Baumann, D., et al. (2018). Exploring cosmic origins with core: Inflation. *Journal of Cosmology and Astroparticle Physics*, 2018(04):016.

Garriga, J., Guth, A. H., and Vilenkin, A. (2007). Eternal inflation, bubble collisions, and the persistence of memory. *Physical Review D*, 76(12):123512.

Guth, A. H. (1981). Inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, 23(2):347.

Guth, A. H. and Pi, S.-Y. (1982). Fluctuations in the new inflationary universe. *Physical Review Letters*, 49(15):1110.

Hawking, S. W. and Ellis, G. F. R. (1973). *The Large Scale Structure of Space-Time.* Cambridge University Press.

Kleban, M. (2011). Cosmic bubble collisions. *Classical and Quantum Gravity*, 28(20):204008.

Kolb, E. W. and Turner, M. S. (1990). *The Early Universe.* Addison-Wesley.

Lee, J. M. (2013). *Introduction to Smooth Manifolds.* Springer, 2 edition.

Liddle, A. R. and Lyth, D. H. (2000). *Cosmological Inflation and Large-Scale Structure*. Cambridge University Press.

Linde, A. D. (1982). A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems. *Physics Letters B*, 108(6):389–393.

Linde, A. D. (1983). Chaotic inflation. *Physics Letters B*, 129(3-4):177–181.

Linde, A. D. (1986). Eternally existing self-reproducing chaotic inflationary universe. *Physics Letters B*, 175:395–400.

Lyth, D. H. and Riotto, A. (1999). Particle physics models of inflation and the cosmological density perturbation. *Physics Reports*, 314:1–146.

Martin, J., Ringeval, C., and Vennin, V. (2014). Encyclopaedia inflationaris. *Physics of the Dark Universe*, 5–6:75–235.

Mattila, P. (1995). *Geometry of Sets and Measures in Euclidean Spaces*. Cambridge University Press.

Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation*. Freeman.

Morgan, F. (2016). *Geometric Measure Theory*. Elsevier, 5 edition.

Mukhanov, V. (2005). *Physical Foundations of Cosmology*. Cambridge University Press.

Munkres, J. R. (2000). *Topology*. Prentice Hall, 2 edition.

Nakahara, M. (2003). *Geometry, Topology and Physics*. CRC Press, 2 edition.

O'Neill, B. (1983). *Semi-Riemannian Geometry with Applications to Relativity*. Academic Press.

Penrose, R. (2004). *The Road to Reality*. Jonathan Cape.

Rudin, W. (1976). *Principles of Mathematical Analysis*. McGraw-Hill, 3 edition.

Schutz, B. (2009). *A First Course in General Relativity*. Cambridge University Press, 2 edition.

Spivak, M. (1979). *A Comprehensive Introduction to Differential Geometry*, volume 1. Publish or Perish.

Starobinsky, A. A. (1980). A new type of isotropic cosmological models without singularity. *Physics Letters B*, 91(1):99–102.

Taylor, J. E. (1076). The structure of singularities in soap-bubble-like and soap-film-like minimal surfaces. *Annals of Mathematics*, 103(3):489–539.

Thurston, W. P. (1997). *Three-Dimensional Geometry and Topology*, volume 1. Princeton University Press.

Tsujikawa, S. (2003). Introductory review of cosmic inflation. *arXiv preprint hep-ph/0304257*.

Vilenkin, A. (1983). The birth of inflationary universes. *Physical Review D*, 27(12):2848–2855.

Wald, R. M. (1984). *General Relativity*. University of Chicago Press.

Weinberg, S. (2008). *Cosmology*. Oxford University Press.

# Appendix

## A Derivation of the Boundary-Driven Expansion Relations

This appendix derives the principal geometric evolution equations presented in the main text. Throughout the derivation, the governing expansion law

$$\frac{dR_i}{dt} = v_0 \Psi_i, \tag{57}$$

is taken as the constitutive relation of the proposed framework, where $v_0 > 0$ denotes the intrinsic maximum radial expansion rate and $\Psi_i \in [0, 1]$ is the free boundary fraction defined in Section 4. The subsequent relations follow directly from elementary differential geometry.

The differential-geometric identities employed throughout this appendix are standard and may be found in do Carmo (1976); Lee (2013).

### A.1 Surface Area Evolution

The surface area of the $i$th universe is

$$S_i(R_i) = 4\pi R_i^2. \tag{58}$$

Applying the chain rule,

$$\frac{dS_i}{dt} = \frac{dS_i}{dR_i}\frac{dR_i}{dt}. \tag{59}$$

Since

$$\frac{dS_i}{dR_i} = 8\pi R_i, \tag{60}$$

substituting Eq. (57) yields

$$\boxed{\frac{dS_i}{dt} = 8\pi R_i v_0 \Psi_i.} \tag{61}$$

Equation (61) shows that the instantaneous rate of surface growth is proportional to both the current radius and the available free boundary.

### A.2 Volume Evolution

The enclosed volume is

$$V_i(R_i) = \frac{4}{3}\pi R_i^3. \tag{62}$$

Differentiating,

$$\frac{dV_i}{dt} = \frac{dV_i}{dR_i}\frac{dR_i}{dt}. \tag{63}$$

Since

$$\frac{dV_i}{dR_i} = 4\pi R_i^2, \tag{64}$$

substitution of Eq. (57) gives

$$\boxed{\frac{dV_i}{dt} = 4\pi R_i^2 v_0 \Psi_i.} \tag{65}$$

Thus, the volume growth is entirely governed by the instantaneous free boundary fraction.

## A.3 Relative Volume Expansion

The relative volume expansion rate is

$$\frac{1}{V_i}\frac{dV_i}{dt}. \tag{66}$$

Substituting Eqs. (62) and (65),

$$\frac{1}{V_i}\frac{dV_i}{dt} = \frac{4\pi R_i^2 v_0 \Psi_i}{\frac{4}{3}\pi R_i^3} \tag{67}$$

$$= \frac{3v_0\Psi_i}{R_i}. \tag{68}$$

Therefore,

$$\boxed{\frac{1}{V_i}\frac{dV_i}{dt} = \frac{3v_0\Psi_i}{R_i}.} \tag{69}$$

This expression demonstrates that the relative expansion rate decreases inversely with the radius when the free boundary fraction remains constant.

## A.4 Limiting Cases

The governing equation immediately yields two important limiting regimes.

If the universe is completely unconstrained,

$$\Psi_i = 1, \tag{70}$$

then

$$\frac{dR_i}{dt} = v_0, \tag{71}$$

which integrates to

$$R_i(t) = R_i(0) + v_0 t. \tag{72}$$

The corresponding surface and volume evolution become

$$\frac{dS_i}{dt} = 8\pi R_i v_0, \tag{73}$$

and

$$\frac{dV_i}{dt} = 4\pi R_i^2 v_0. \tag{74}$$

Conversely, if the entire boundary becomes constrained,

$$\Psi_i = 0, \tag{75}$$

then

$$\frac{dR_i}{dt} = 0, \tag{76}$$

implying

$$\frac{dS_i}{dt} = 0, \qquad \frac{dV_i}{dt} = 0. \tag{77}$$

These limiting cases establish the admissible range

$$0 \leq \frac{dR_i}{dt} \leq v_0, \tag{78}$$

which follows directly from

$$0 \leq \Psi_i \leq 1. \tag{79}$$

Consequently, the proposed framework defines a bounded geometric dynamical system in which all expansion rates are determined solely by the available free boundary fraction.